NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1029

A CONDITION ON THE INITIAL SHOCK

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SUMMARY

Initial shocks of the type that occur on airfoils at stream Mach numbers less than unity are shown to satisfy a certain condition, namely, that the local Mach number behind the shock wave tends to approach unity. This result is, in nature, similar to the classic condition of Kutta on the circulation.

INTRODUCTION

When the flow velocity on the surface of an airfoil exceeds the local velocity of sound, there is a local restricted supersonic potential flow field bounded by the contour on one side and by the line $M = 1$ on the remainder. Where a certain maximum supersonic velocity is attained, as in the middle of a section with fore-and-aft symmetry, there will be a gradual reduction in velocity and the supersonic region will merge into the subsonic field. When the critical condition is reached, there will be a rather sudden change in the flow. Instead of reverting back to subsonic by a gradual and potential flow, the velocity will not reach a maximum in the central point as before but will increase farther along the contour and then revert to sonic value (locally) by a sudden shock. This shock causes a corresponding (sudden) increase in the drag and a reduction in the circulation. The velocities will for this reason be considerably below the Prandtl-Meyer values. In the following discussion it will be shown by a simple argument that the mission of the shock is essentially that of bringing the local velocity back to a local sonic value, that a smaller shock is insufficient, and that a greater shock cannot be accommodated.

THEORY

The shock relations derived by Meyer and Prandtl (reference 1) may be given as

$$u_{1n}\, u_{2n} = c_s^2 - \frac{\kappa - 1}{\kappa + 1} u_t^2$$

and

$$u_t = u_{1t} = u_{2t}$$

where u_{1n} and u_{2n} are the normal components of the velocity and u_{1t} and u_{2t} are the tangential components of the velocity before and after a shock wave, respectively; the quantity c_s is the critical velocity of sound; and κ is the adiabatic constant.

By use of the local Mach number $M = \frac{V}{c}$ where V is the local fluid velocity and c is the local velocity of sound, the following simple and symmetric relations are obtained:

$$\frac{M_{1t}^2}{1 + \frac{\kappa - 1}{2} M_1^2} = \frac{M_{2t}^2}{1 + \frac{\kappa - 1}{2} M_2^2}$$

$$\frac{M_1^2}{1 + \frac{\kappa - 1}{2} M_1^2}\, \frac{M_2^2}{1 + \frac{\kappa - 1}{2} M_2^2} = \left(\frac{2}{\kappa + 1}\right)^2$$

where M_1 and M_{1t} are the normal and tangential components of the Mach number on the upstream side and M_2

and M_{2t} are the same quantities on the downstream side of the shock line. (See fig. 1.)

The relations for the angle of deflection δ are

$$\cot \delta = \frac{\sqrt{(A_1 + C)(A_2 + C)} + B - C}{\sqrt{(A_1 + C)(B - C)} - \sqrt{(A_2 + C)(B - C)}}$$

and

$$\cos \delta = \frac{\sqrt{(A_1 + C)(A_2 + C)} + B - C}{\sqrt{D_1 D_2}}$$

where

$$A_1 = M_{2T}^2\left[2(\kappa - 1)^2 - (\kappa + 1)^2\right] + M^2(\kappa + 1)^2 + 4(\kappa - 1)$$

$$A_2 = M^2\left[2(\kappa - 1)^2 - (\kappa + 1)^2\right] + M_{2T}^2(\kappa + 1)^2 + 4(\kappa - 1)$$

$$B = 4\kappa(\kappa - 1)M^2 M_{2T}^2 + \left(M^2 + M_{2T}^2\right)\left(-\kappa^2 + 6\kappa - 1\right) - 4(\kappa - 1)$$

$$C = \pm(\kappa + 1)\sqrt{\left(M^4 + M_{2T}^4\right)(\kappa + 1)^2 + 2M^2 M_{2T}^2\left(\kappa^2 - 6\kappa + 1\right) + 8(\kappa - 1)\left(M^2 + M_{2T}^2\right) + 16}$$

$$D_1 = 4\kappa M^2\left[(\kappa - 1)M_{2T}^2 + 2\right]$$

$$D_2 = 4\kappa M_{2T}^2\left[(\kappa - 1)M^2 + 2\right]$$

and M and M_{2T} are the (total) Mach numbers on the upstream and downstream sides of the shock line, respectively.

The relation for the angle of the shock line α is

$$\cos^2\alpha = \frac{A_1 + C}{M^2 D}$$

where $D = \frac{D_1}{M^2}$. It may further be shown that the pressure ratio

$$\frac{p_2}{p_1} = \frac{2\kappa}{\kappa + 1} \frac{A_1 + C}{D} - \frac{\kappa - 1}{\kappa + 1}$$

and the temperature ratio

$$\frac{T_2}{T_1} = \frac{\left[\kappa\left(A_1 + C\right) - \frac{\kappa - 1}{2} D\right]\left[\frac{\kappa - 1}{2}\left(A_1 + C\right) + D\right]}{D\left(\frac{\kappa + 1}{2}\right)^2\left(A_1 + C\right)}$$

The angle α for $M_{2T} = 1$ is given by the expression

$$\cos^2\alpha = \frac{M^2 - \frac{3 - \kappa}{\kappa + 1} \pm \sqrt{M^4 - 2M^2 \frac{3 - \kappa}{\kappa + 1} + \frac{(3 - \kappa)^2 + 16\kappa}{(\kappa + 1)^2}}}{\frac{4\kappa}{\kappa + 1} M^2}$$

For large values of M

$$\cos^2\alpha \rightarrow \frac{\kappa + 1}{2\kappa}$$

For $\kappa = 1.4$

$$\cos^2\alpha \rightarrow \frac{6}{7}$$

The relationships among the various variables are shown in figures 1 to 7. Figure 1 shows the relations of M, M_1, M_2, M_{1t}, M_{2t}, M_{2T}, α, and δ. Cos δ is plotted against M for various values of M_{2T} in figure 2 and the angle δ is plotted similarly in figure 3, which also shows a magnified plot near $M = 1$ for $M_{2T} = 1$. Figures 4 and 5 show the pressures and temperatures, respectively, resulting from shocks with $M_{2T} = 1$. The variation of $\cos^2\alpha$ with M for $M_{2T} = 1$ is shown in figure 6. In this figure the asymptotic value should be noted. In figure 7 the function $M^2\cos^2\alpha$ is plotted against M for $M_{2T} = 1$.

Values of $\cos\delta$ and δ are listed in table I for various values of M and M_{2T}.

DISCUSSION

The nature of the oblique shock for $M_{2T} = 1$ will now be indicated. Suppose for the moment that the shock on the upper side of an airfoil results in a value $M_{2T} > 1$. The flow is then still supersonic and, since the flow diverges on the back part of the airfoil, the Mach number will increase until a second shock occurs. This second shock may or may not produce a subsonic region; the process therefore continues until finally a local Mach number of unity is reached behind the last shock. From then on no further shocks will occur since in a diverging flow the subsonic velocity decreases.

If only a single shock is permitted, it would be expected to replace the multiple shock and produce a Mach number of unity behind the shock front.

If the shock should exceed such a shock in intensity and cause a subsonic velocity behind the shock front, this shock will result in a greater entropy increase and a greater drag than the corresponding values of the shock previously defined. Nature will prefer the shock that gives the smaller entropy increase; hence the shock will reduce the velocity to a sonic value but no further.

The argument here differs from that used by Tsien and Fejer (reference 2), who favor the condition of maximum deflection as the criterion. Dailey (reference 3) chooses a pressure coefficient behind the shock that is numerically equal to the pressure coefficient at the local velocity of sound ahead of the wave. Tsien's numerical result agrees closely with those in this paper.

CONCLUSION

A tentative theory is given for a special condition on the initial shock wave, which fixes a preferred shock as the one resulting in a local sonic velocity behind the shock front.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., September 4, 1945

REFERENCES

1. Ackeret, J.: Gasdynamik. Handbuch der Physik, Bd. VII, Kap. 5, Julius Springer (Berlin), 1927, pp. 330, 331.

2. Tsien, Hsue-shen, and Fejer, Andrej: A Method for Predicting the Transonic Flow over Airfoils and Similar Bodies from Data Obtained at Small Mach Numbers. GALCIT Rep., Dec. 31, 1944.

3. Dailey, C. L.: Prediction of Pressure Distribution at Sub-Critical and Super-Critical Mach Numbers. Rep. No. 345, Douglas Aircraft Co., Inc., May 24, 1943.

TABLE I

VALUES OF $\cos\delta$ AND δ

$\cos\delta$

M_{2T} \ M	0.25	0.50	0.75	1.00	1.50	2.00	3.00	4.00	5.00
0.25	1.0000	1.2036		1.6330		1.6920	1.5763	1.5059	1.4660
.50	1.2036	1.0000	1.0441	1.0953	1.1086	1.0616	.9740	.9251	.8981
.75		1.0441	1.0000	1.0078		.9351	.8465	.7995	
1.00	1.6330	1.0953	1.0078	1.0000	.9792	.9225	.8290	.7798	.7535
1.50		1.1086		.9792	1.0000	.9717	.8781	.8234	.7938
2.00	1.6920	1.0616	.9351	.9225	.9717	1.0000	.9403	.8837	.8515
3.00	1.5763	.9740	.8465	.8290	.8781	.9403	1.0000	.9702	.9383
4.00	1.5059	.9251	.7995	.7798	.8234	.8837	.9702	1.0000	.9848
5.00	1.4660	.8981		.7535	.7938	.8515	.9383	.9848	1.0000

δ, deg

M_{2T} \ M	1.00	1.10	1.20	1.30	1.40	1.50	2.00	3.00	4.00	5.00
0.25										
.50								13.09	22.32	26.09
.75							20.76	32.17	36.92	
1.00	0	1.40	3.70	6.32	9.03	11.70	22.71	34.00	38.76	41.11
1.50						0	13.66	28.59	34.57	37.46
2.00							0	19.90	27.91	31.62
3.00								0	14.02	20.23
4.00									0	10.00
5.00										0

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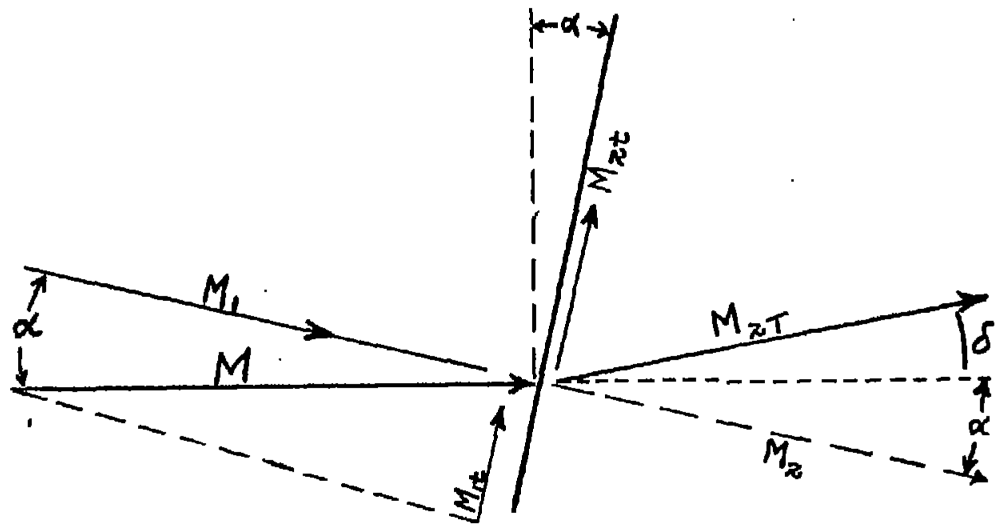

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Figure 1.- Relations among M, M_1, M_2, M_{1t}, M_{2t}, M_{2T}, α, and δ.

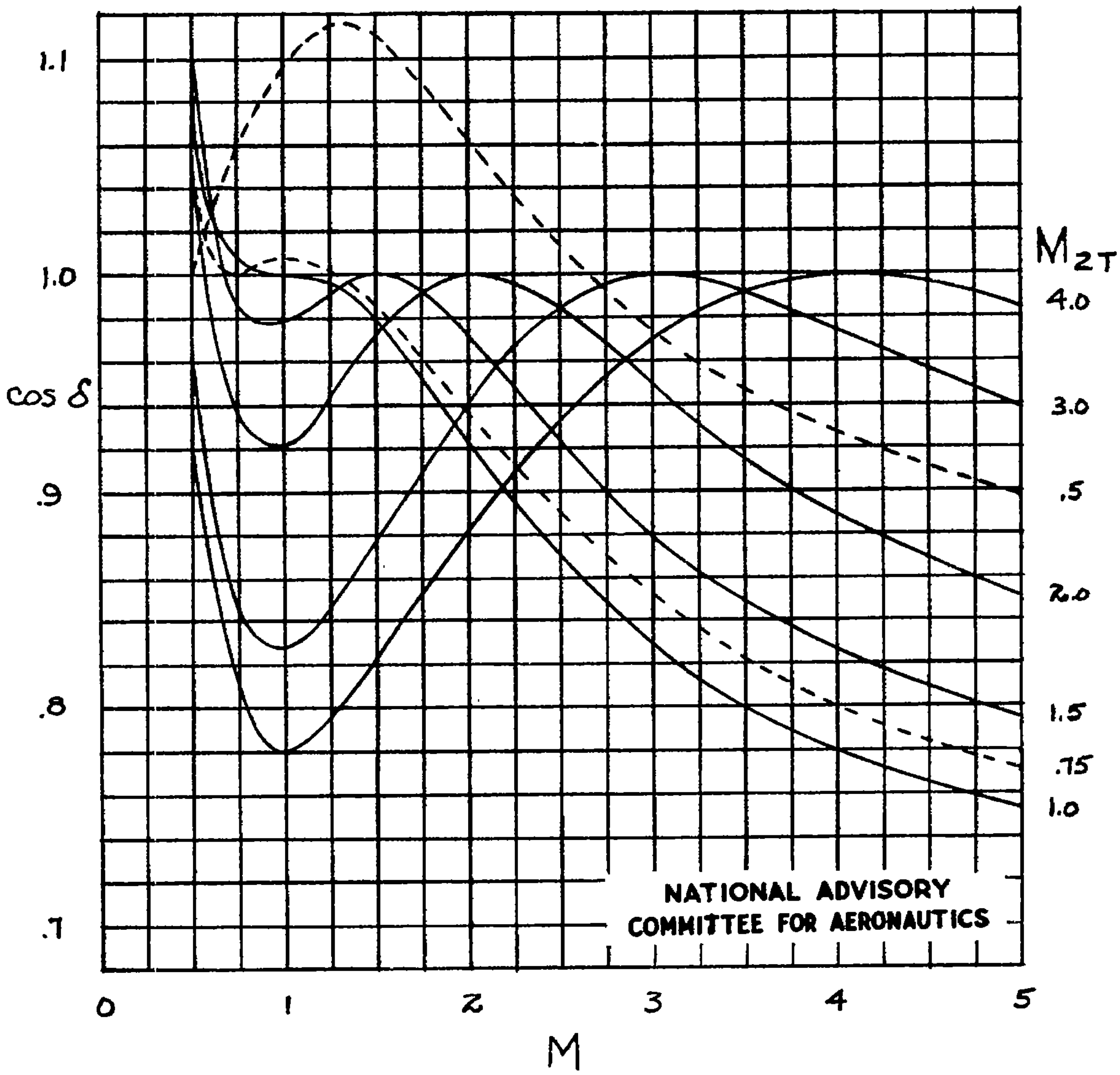
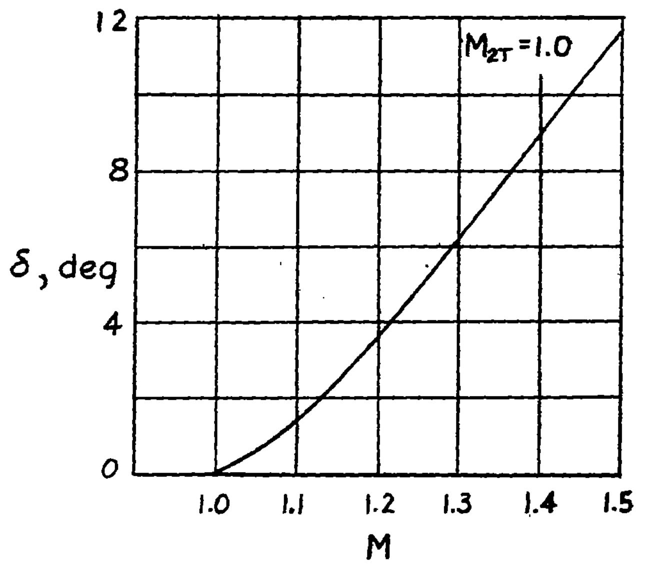


Figure 2.- Variation of cos δ with M for various values of M_{2T}.



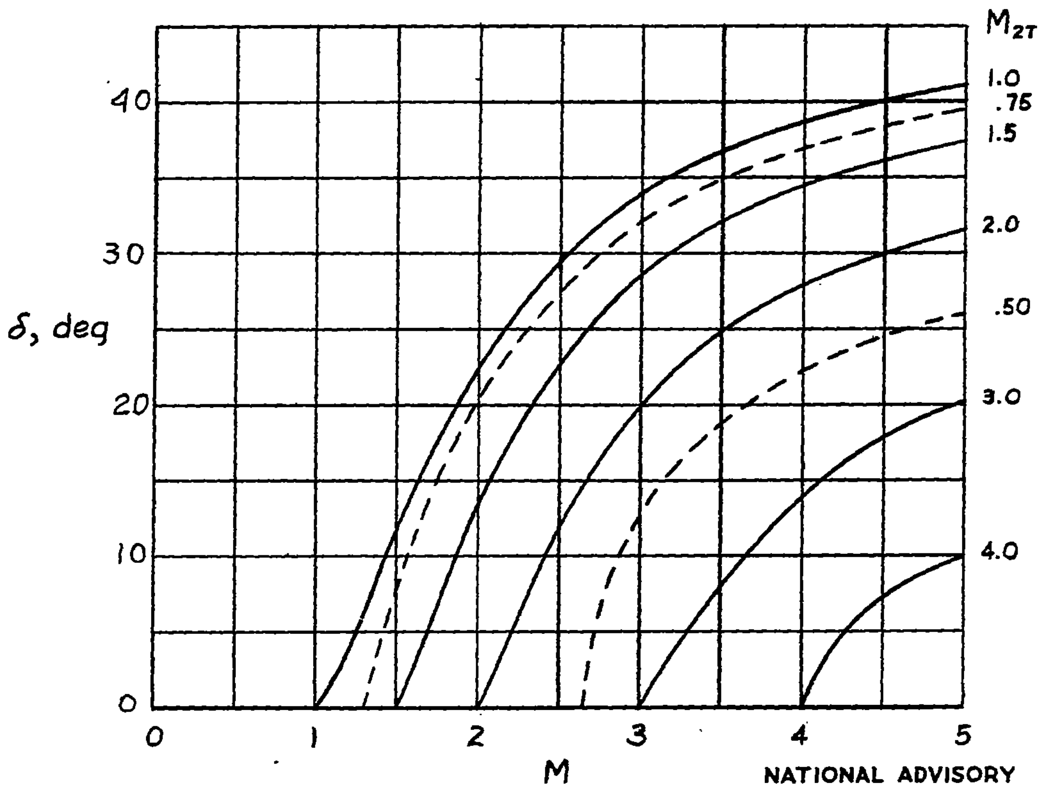


Figure 3.- Variation of δ with M for various values of M_{2T}. Portion near M = 1 for M_{2T} = 1 magnified.

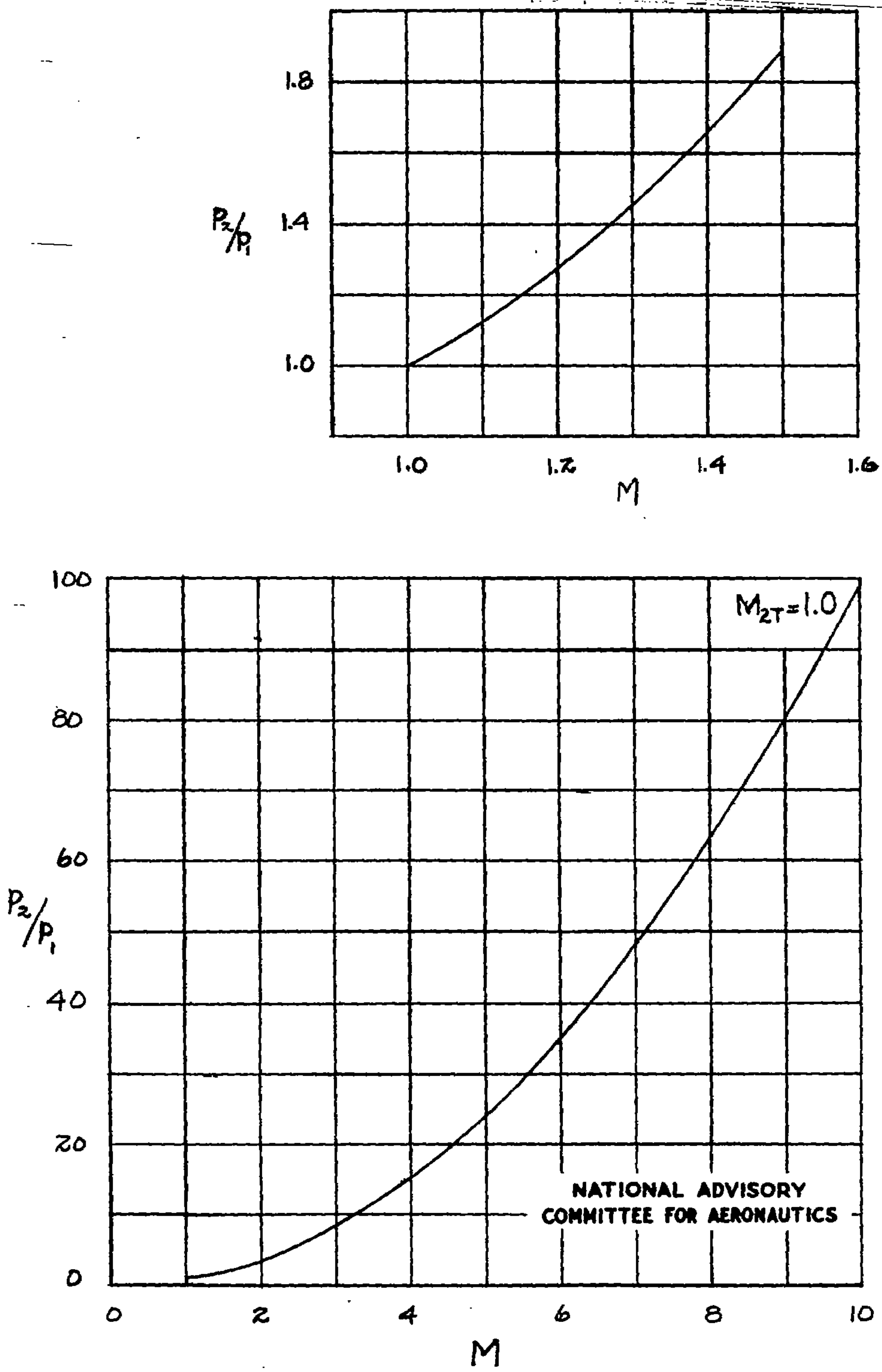


Figure 4.- Variation of ratio of pressures resulting from shocks with M for $M_{2T} = 1$.

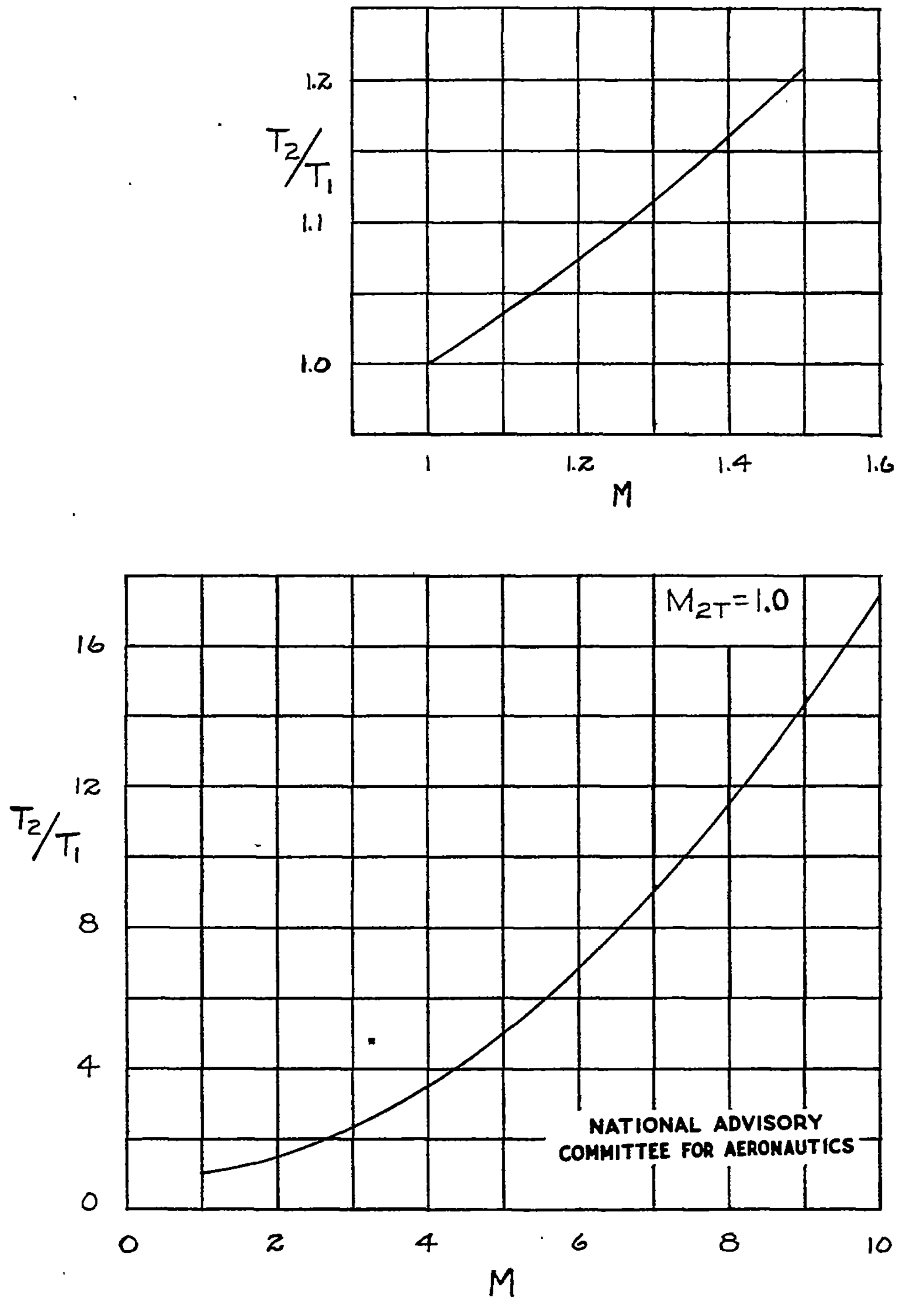


Figure 5.- Variation of ratio of temperatures resulting from shocks with M for M_{2T} = 1.

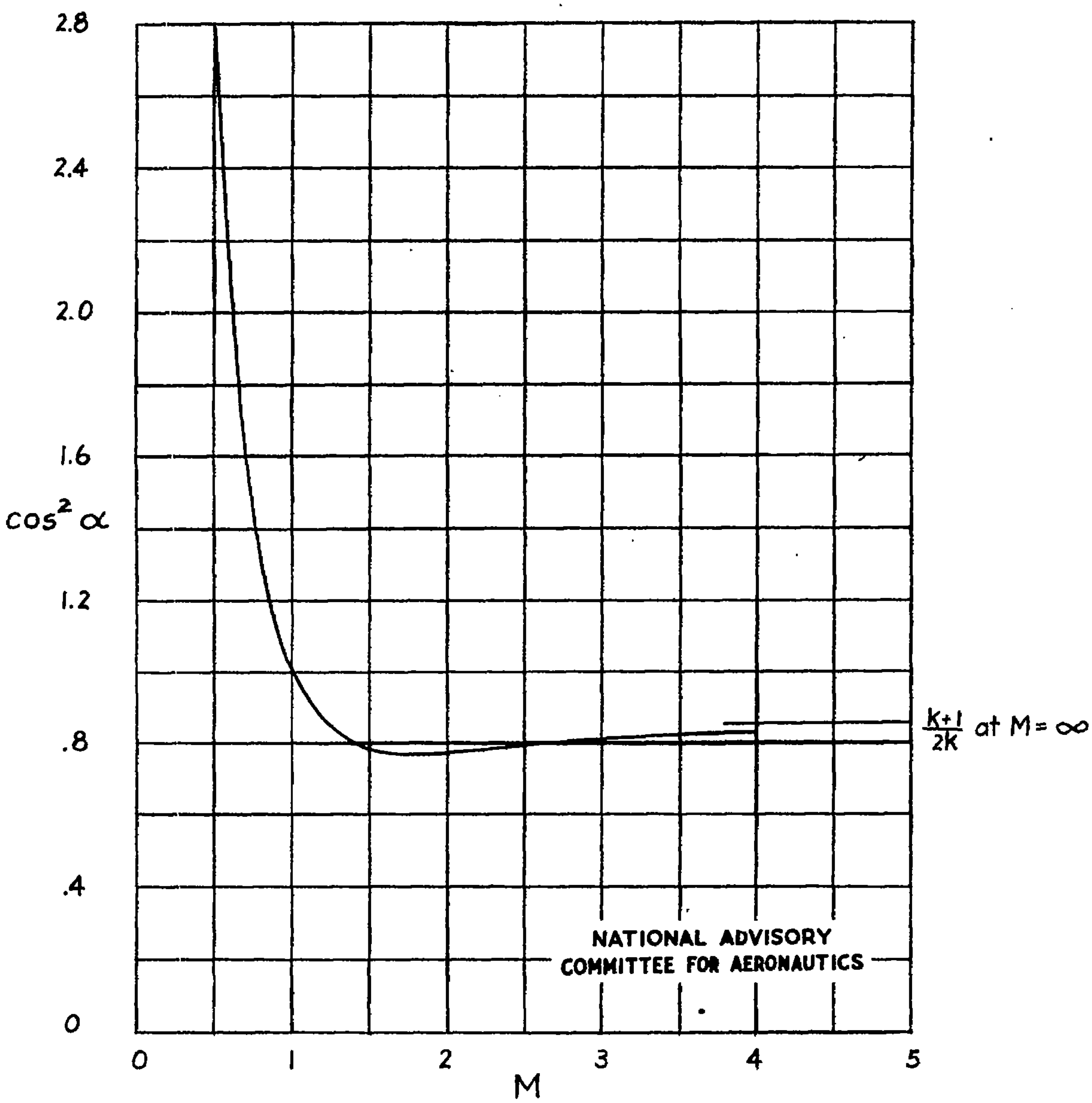


Figure 6.- Variation of $\cos^2\alpha$ with M for $M_{2T} = 1$.
(Note the asymptotic value.)

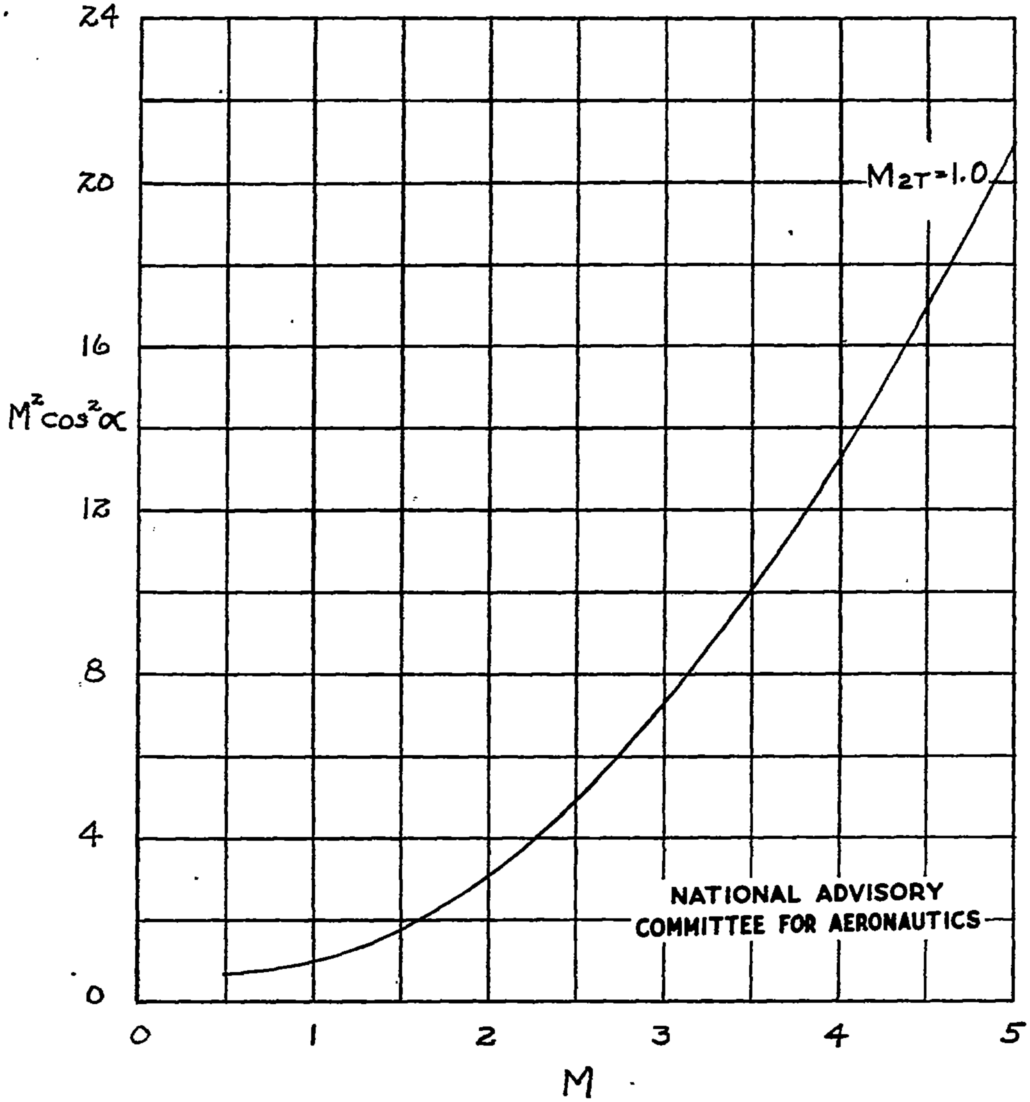


Figure 7.- Variation of function $M^2\cos^2\alpha$ with M for $M_{2T} = 1$.